10/6/17 "Its always too early to quit." -Norman Peale

HW: ?

Test 2 on Monday 10/16

AIM: What are some basic rules for Differentiation?

Warm Up:

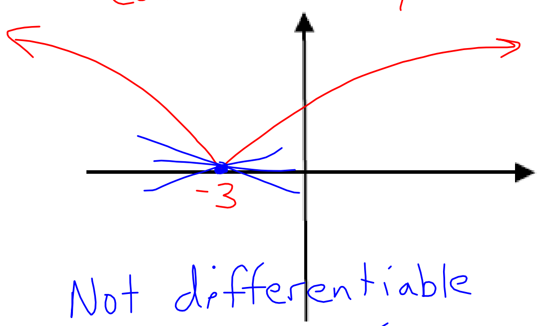
## Differentiability Implies Continuity:

If $f$ is differentiable at $x = c$, then $f$ is continuous at $x = c$.

1. If a function is differentiable at $x = c$, then it is continuous at $x = c$. So, **differentiability implies continuity**.
2. It is possible for a function to be continuous at $x = c$ and not be differentiable at $x = c$. **Continuity does not imply differentiability**.

$f(x) = (x+3)^{\frac{2}{3}}$

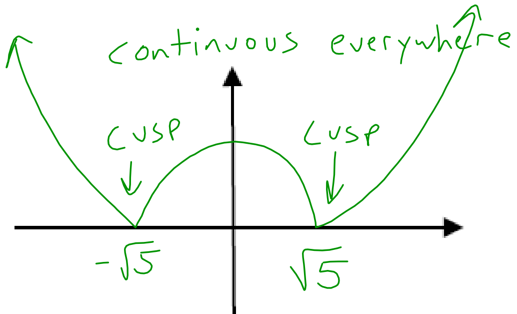
continuous everywhere




Not differentiable @ $x = -3$ (No derivative)

(cusp)

$g(x) = |x^2 - 5|$




Not differentiable @ $x = \pm\sqrt{5}$

(*) Functions are not differentiable (No derivative) wherever there is cusp, corner, break, or vertical tangent line.

# Basic Differentiation Rules

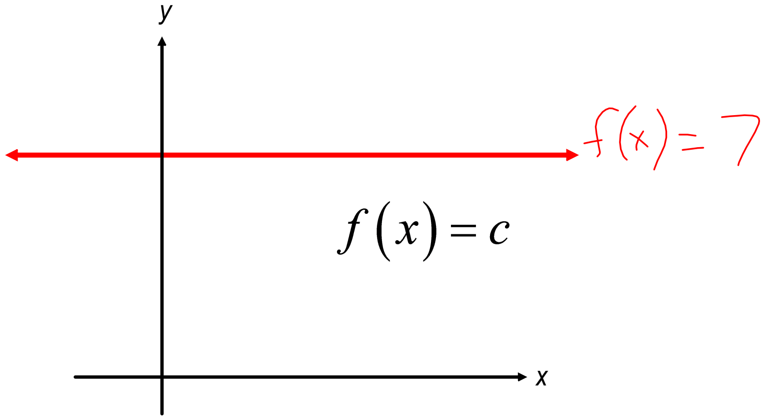
To find Derivatives

## 1. The Constant Rule

The derivative of a constant function is 0.

For any real number, $c$ : $\frac{d}{dx}[c] = 0$

The slope of a horizontal line is 0.




The derivative of a constant function is 0.

A.) $y = -4$

$y' = 0$

B.) $s(t) = 45$

$s'(t) = 0$

C.) $f(x) = 0$

$f'(x) = 0$

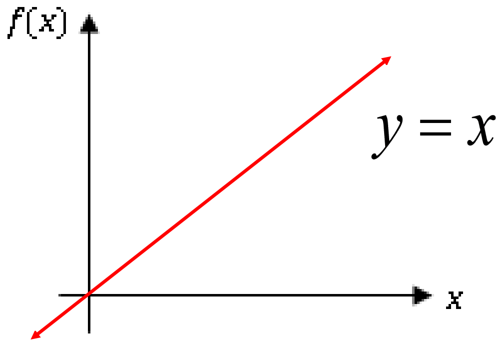
D.) $y = \frac{k\pi}{2}$

$\frac{dy}{dx}$ or $y' = 0$

## 2. The Single Variable Rule:

The derivative of *x* is 1. $\frac{d}{dx}[x]=1$

Think graphically about the line *y* = x.




A.) $f(x)=x$ B.) $s(t)=t$ C.) $x(t)=t$

$f'(x)=1$ $s'(t)=1$ $x'(t)=1$

## 3. The Power Rule

If $n$ is a rational number then the function $x^n$ is differentiable and $\frac{d}{dx}\left[x^n\right] = n \bullet x^{n-1}$

**EX #3:** Use the power rule to find the derivatives of the following, if:

A.) $y = x^2$

$y' = 2x^{2-1}$

$\boxed{y' = 2x}$

B.) $f(x) = x^4$

$\boxed{f'(x) = 4x^3}$

C.) $y = \sqrt{x} = x^{\frac{1}{2}}$

$y' = \frac{1}{2}x^{-\frac{1}{2}}$

$y' = \frac{1}{2x^{1/2}}$

$\boxed{y' = \frac{1}{2\sqrt{x}}}$

D.) $g(x) = \frac{1}{x} = x^{-1}$

$g'(x) = -1x^{-2}$

$g'(x) = \frac{-1}{x^2}$

E.) $f(x) = \frac{1}{x^2} = x^{-2}$

$f'(x) = -2x^{-3}$

$f'(x) = \frac{-2}{x^3}$

F.) $y = \frac{1}{x^{2/3}} = x^{-\frac{2}{3}}$

$y' = -\frac{2}{3}x^{-5/3}$

$y' = \frac{-2}{3x^{5/3}} = \boxed{\frac{-2}{3\sqrt[3]{x^5}}}$

## 4. The Constant Multiple Rule:

The derivative of the term $ax^n$, where *a* and *n* are real numbers, is $(a \bullet n)x^{n-1}$

STEPS:

1. Multiply the coefficient by the variable's exponent. If no coefficient is stated -- in other words, the coefficient equals 1-- the exponent becomes the new coefficient.
2. Subtract 1 from the exponent.

**EX #4:** Use the constant multiple rule to find the derivatives of the following, if:

A.) $y = 3x^2$

$y' = 6x^1$

$\boxed{y' = 6x}$

B.) $s(t) = -5t^3$

$\boxed{s'(t) = -15t^2}$

C.) $y = 6\sqrt{x} = 6x^{\frac{1}{2}}$

$y' = 3x^{-\frac{1}{2}}$

$y' = \frac{3}{x^{1/2}} = \frac{3}{\sqrt{x}}$

D.) $g(x) = \frac{3}{x^2} = 3x^{-2}$

$g'(x) = -6x^{-3}$

$\boxed{g'(x) = \frac{-6}{x^3}}$

E.) $f(x) = \frac{4}{\sqrt{x}}$

F.) $y = \frac{-8}{\sqrt[3]{x^4}} = -8x^{-\frac{4}{3}}$

$y' = \frac{32}{3}x^{-7/3}$

$y' = \frac{32}{3x^{7/3}} = \boxed{\frac{32}{3\sqrt[3]{x^7}}}$

## 5. The Sum and Difference Rules:

The derivative of a sum or difference is the sum or difference of the derivatives.

Take the derivative of each piece

$$\frac{d}{dx}\left[f(x)+g(x)\right]=f'(x)+g'(x)$$

and

$$\frac{d}{dx}\left[f(x)-g(x)\right]=f'(x)-g'(x)$$

A.) $f(x)=x^3-5x^2+2x-7x^0$

$f'(x)=3x^2-10x^1+2x^0-0$

$$\boxed{f'(x)=3x^2-10x+2}$$

B.) $y=\frac{2}{5}x^5+x^4+3x^2-1$

$$\boxed{y'=2x^4+4x^3+6x}$$

C.) $g(x) = \sqrt[3]{x} + 5\sqrt{x}$

$$g(x) = x^{\frac{1}{3}} + 5x^{\frac{1}{2}}$$

$$g'(x) = \frac{1}{3}x^{-\frac{2}{3}} + \frac{5}{2}x^{-\frac{1}{2}}$$

$$\boxed{g'(x) = \frac{1}{3\sqrt[3]{x^2}} + \frac{5}{2\sqrt{x}}}$$

D.) $y = 5x^{3/2} - 6x^{5/3} + \frac{4}{x^{1}}$

$$y = 5x^{3/2} - 6x^{5/3} + 4x^{-1}$$

$$y' = \frac{15}{2}x^{\frac{1}{2}} - \frac{30}{3}x^{\frac{2}{3}} - 4x^{-2}$$

$$\boxed{y' = \frac{15}{2}\sqrt{x} - 10\sqrt[3]{x^2} - \frac{4}{x^2}}$$

E.) $h(x) = 3x^2 - \dfrac{4}{x^2} + 2\sqrt{x}$

F.) $f(x) = \dfrac{8x^3 + 4x^2 - 3}{x}$

$f(x) = \dfrac{8x^3}{x} + \dfrac{4x^2}{x} - \dfrac{3}{x}$

$f(x) = 8x^2 + 4x - 3x^{-1}$

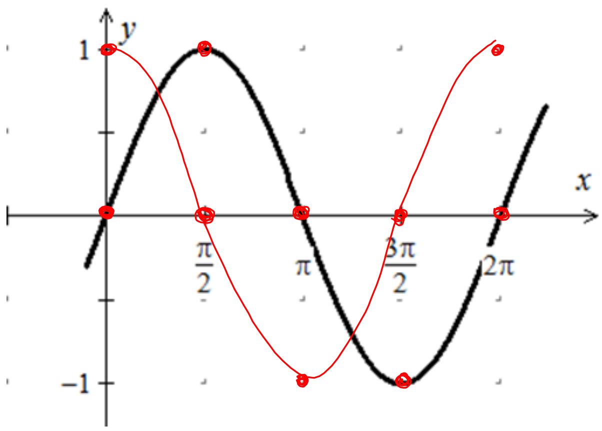
$f'(x) = 16x + 4 + 3x^{-2}$

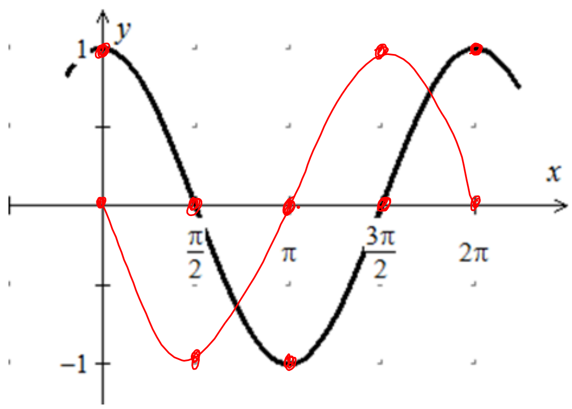
$f'(x) = 16x + 4 + \dfrac{3}{x^2}$

## 6. Derivatives of Sine and Cosine Functions:

$$\frac{d}{dx}[\sin x] = \cos x \qquad \text{and} \qquad \frac{d}{dx}[\cos x] = -\sin x$$

EX #4: The derivative of the sine function is the cosine



EX #5: Find the derivative of the sine function by using the limit process.